A Diffusion Model for Highly Fluctuating Phenomenon: Application to Global Hunger

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Abstract

Diffusion models are highly studied in physics but are seldom explored in social sciences. However, many social phenomena can be succinctly described in terms of sociological phenomenon such as imitation, cohesion, and group function, which are all characteristics of a diffusion model. In this paper, a diffusion model of the Fick's type is proposed to describe the underlying behavior of hunger spread in different countries. Based on the global hunger indices of 69 countries from 2008, 2013, and 2018, the indices were decreasing from 2008 to 2013. However, towards 2018 the hunger indices rose. With the Fick's type diffusion model of hunger, its spread in different countries seem to be irregular. As time passes the hunger indices will be distributed in a normal curve. With these results, hunger continually persists, even with the presence of anti-hunger interventions. The behavior of hunger incidence depends on the country's living condition and environment. War, drought, and even diet may cause hunger. Sustaining the decrease of hunger indices of a country is a growing concern that every country faces nowadays.

Keywords: Global hunger, complex adaptive system, diffusion model, phenomenon

Introduction

Mathematical models are used to explain occurrences of complex social phenomenon. One of the models known is on diffusion. Diffusion models have been studied and developed to describe the spread of social phenomena (Rogers, 1962; Palloni, 2001). Hunger, as a social phenomenon (Wells, Miller, & Deville, 1983), can thus be analyzed within the context of diffusion theory

(Jaakkola, 1996). In social sciences, models of diffusion processes have been explored in different fields like the transmission of political opinions in political science, the spread of innovations in management science, and the spread of smoking behaviors in public healthcare. These diffusion processes are complex systems that are mathematically modelled in an attempt to describe the extent to which these phenomena spread. In this paper, a diffusion model of Fick's type is proposed to describe the spread of hunger in different countries.

In its physical sense, Fick's type of diffusion is defined as a phenomenon where a certain particle group as a whole spreads according to the irregular motion of each particle (Kandler & Unger, 2010). The spread is always directed from regions of higher concentration to regions of lower concentration and the time dependence of the distribution of the particles in space is given by the so-called diffusion equation, which is the mathematical formulation of the described spread dynamics. This type of diffusion is most fitted because the processes of spread do not always involve the adoption of new behaviors, but may include neglect of a recently adopted behavior or resistance to change (Crank, 1975; Palloni, 2001).

This type of diffusion theory seeks to explain the wide spread behavior of a group of particles (rather than the spread behavior of a single particle), and consequently the variable of interest is the proportion of the particle group which can be found in location x at time t. In this way phenomena like the diffusion of hunger in the global scenario can be explained, with the idea that the rate of change of population getting hungry is directly proportional to how fast the rate of change of the population is changing.

Global hunger phenomenon has been studied by various researchers. Although hunger is being studied worldwide, not much of those attempted to model the diffusion processes of hunger incidence in the different countries around the globe. At present, the world is characterized by the coexistence of agricultural abundance and prevalent hunger (Webb, Stordalen, Singh, Shetty, & Lartey, 2018). Webb and colleagues (2018) cited that even there is a bounty of food output globally, hunger is still present. Despite movements of eradicating hunger, there are still 821 million people going to bed with an empty stomach every night (World Food Programme, 2018).

Conceptual Framework

One common framework for investigating any problem in pattern formation involves the use of systems of diffusion. In 1855, physiologist Adolf Fick first reported his now well-known laws governing the transport of mass through diffusive means. Fick's second law predicts how diffusion causes the concentration to change with respect to time. It is a partial differential equation in the form,

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2}$$

where P represents the dimension concentration; t denotes time; D symbolizes diffusion coefficient in dimensions; and signifies position or length. In the study, the equation reflects the rate of change of population getting hungry ($\frac{\partial P}{\partial t}$) is directly proportional to the rate of change to how fast the rate of change of the population is changing ($D\frac{\partial^2 P}{\partial x^2}$ with the same diffusion coefficient.

In 1952, Turing applied such a framework to understand the fundamental problem in developmental biology, where he introduced the concept of morphogens. This substance is thought to be involved in the patterning of cells during embryonic developments, and by diffusion, it could determine the development of cells which would respond differently to diverse concentrations. He used the law of diffusion to model the movement of morphogens from the regions or cells with a higher concentration to the region with a lower concentration.

Social scientists make use of the diffusion laws to describe the process of change, which attempts to predict the behavior of individuals and social groups in the process considering different variables (Padel, 2001). Rogers' (1962) diffusion of innovation (DOI) theory, one of the oldest social science theories, explains how, over time, a behavior gains momentum and spreads through a specific population. The outcome of this diffusion is that people, as part of a social system, adopt a new behavior. Adoption means that people do something differently from what they did previously, which is possible through diffusion.

This diffusion theory has been used successfully in many fields, including communication, agriculture, public health, criminal justice, social work, and marketing. In terms of hunger phenomenon, Garlasco and colleagues (2019) developed a hierarchical drift-diffusion model to compute the relative influence of hunger, caloric density, and valence on food choice. Findings revealed the complex nature of food choices and the usefulness of nuanced computational models to address the multifaceted nature of decision-making and value assessment processes affecting food selection. Results indicated that hunger, caloric density, and valence affected how fast participants accumulated information in favor of the chosen item over the other. In addition, a bio-inspired motivational model was developed by Zamarripa and colleagues (2015) to generate, maintain, and dismiss needs related to hunger. Results of the implementation confirmed that the model responds as expected, and if other diverse activities are integrated, behaviors are more diverse.

Methodology

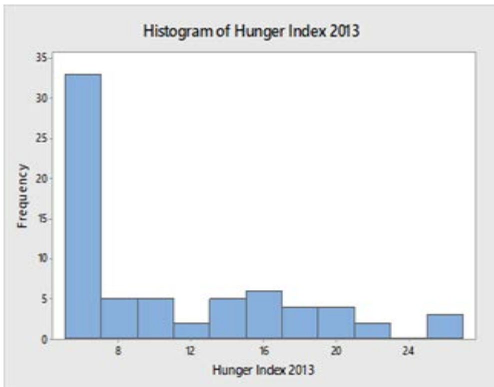
This study utilized the complex adaptive system approach through diffusion models in analyzing the hunger phenomenon. Complex Adaptive Systems (CAS) is composed of a large number of components called agents that interact and adapt or learn (Holland, 2006). These agents in the context of this study are the persons who experienced hunger in different countries measured by the 2018 Global Hunger Index (GHI). The global hunger index ranks countries on a 100-point scale, with 0 being the best score (no hunger) and 100 being the worst, although neither of these extremes is reached in practice. Values from 0 to 9.9 reflect low hunger, values from 10.0 to 19.9 reflect moderate hunger, values from 20.0 to 34.9 indicate serious hunger, values from 35.0 to 49.9 reflect alarming hunger, and values of 50.0 or more reflect extremely alarming hunger levels.

International non-governmental organizations such as Welthungerhilfe and Concern Worldwide, together with the International Food Policy Research Institute (IFPRI), developed a tool to measure hunger globally since 2006. The global hunger index (GHI) presents a multidimensional measure of global hunger capturing three dimensions: insufficient availability of food, shortfalls in the nutritional status of children, and child mortality. Data sets of these dimensions were taken from the Food and Agriculture Organization of the United Nations (FAO), the World Health Organization (WHO), and United Nations Children's Fund (UNICEF).

CAS is a viable method for modelling complex physical and social systems to understand their behavior based on observed data (Kaisler & Madey, 2008). Partial differential equations (PDEs) have been employed by researchers to effectively model and simulate complex adaptive systems. These are reduction based approaches which develop models, namely diffusion models, to view the system globally, explaining how, in the course of time, a behavior gains momentum and spreads through a specific population.

Results and Discussions

Figure 1 presents the histogram of countries' hunger indices in 2008, 2013, and 2018. As shown in the graphs, hunger incidences vary in different countries from the given years. From 2008 to 2013, the histogram of hunger indices in 2013 presents improvement in declining hunger occurrences as compared to the 2008 results. However in 2018, it is evident in the histogram that hunger incidence worsens and becomes widespread in different countries in the given period of time.



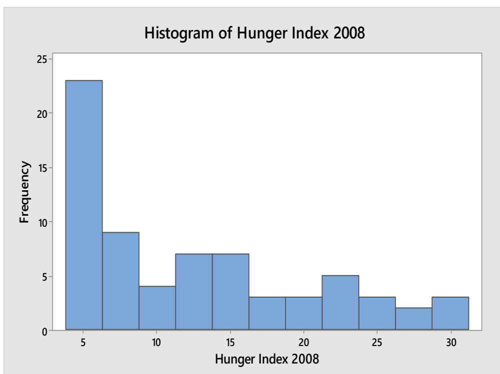


Figure 1. Histogram of Global Hunger Indices of countries in 2008, 2013 and 2018.

The complexity of hunger phenomenon is clearly reflected in the results. The movement of different countries' hunger indices on the given time reflects that the behavior of hunger is complicated and is difficult to understand. There are so many agents that interact in various anti-hunger strategies of countries which make it difficult to predict hunger occurrence simply by looking at the individual interactions.

In 2008, 33 countries had levels of hunger that were alarming or extremely alarming. According to Von Grebmer and colleagues (2008), the year's global hunger index reflected high levels of hunger, mostly in Sub-Saharan Africa, while South Asia has made fast improvement in combating hunger. The rising food prices pose serious threats where people can afford even less. After five years, the 2013 index indicated that global hunger fell from the 2008 score. However, Von Grebmer and party (2013) reported that world hunger remains serious, with 19 countries suffering from levels of hunger that are alarming to extremely alarming. South Asia has the highest regional hunger score followed by Sub-Saharan Africa. It was pointed out that social inequality and the low nutritional, educational, and social status of women are major causes of child under-nutrition in the South Asian region.

The 2018 hunger scores for South Asia and Sub-Saharan Africa had higher indices than those of other regions of the world. Von Grebmer et al. (2018) cited that their scores indicate serious levels of hunger, while other regions have low or moderate hunger levels. However, even those regions with low or moderate hunger scores include countries where hunger and undernourishment are problematically high like Haiti and Yemen. Of the countries with moderate, serious, alarming, or extremely alarming hunger levels, 16 countries have no improvement or even experienced deterioration in hunger levels since 2008. This could be due to climate change, which triggers aberrations in temperature and more natural calamities (typhoons, flooding) that have caused damage to agricultural production, soil erosion, and others. These environmental changes could be factors that would be attributed to the changes in scores and indices.

To fully explain the behavior of the global hunger phenomenon, a diffusion model anchored on Fick's second law of diffusion is derived. Starting from the partial differential equation in the form,

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2} \quad (1)$$

where $\frac{\partial P}{\partial t}$ reflects the rate of change of the proportion of hungry people and $\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2}$ describes the rate of change to how fast the rate of change of the population is changing with the same diffusion coefficient.

The partial differential equation has boundary conditions as follows:

$P(0,t) = 0$, given x = 0 at any time t , the function is also 0. (2)

$P(L,t) = 0$, given x = L (length) at time t, the function is 0. (3)

$P(x,0) = f(x)$ given t = 0 at any x, it is the density function. (4)

Assume that $P(x,t) = X(x) \cdot T(t)$, that is, location *x* is independent to time *t*. (5)

Obtaining the first and second partial derivatives of (5), we have

$$\frac{\partial P}{\partial t} = X(x) \cdot T'(t) \quad (6)$$

$$\frac{\partial^2 P}{\partial x^2} = X''(x) \cdot T(t) \quad (7)$$

Substituting (5) to (1), it resulted in

$$X(x) \cdot T'(t) = D \cdot X''(x) \cdot T(t) \quad (8)$$

Thus,

$$\frac{T'(t)}{DT(t)} = \frac{X''(x)}{X(x)} = k \quad (9)$$

From (9), the following is derived:

(i) $\frac{T'(t)}{DT(t)} = k$ implies that $\int d(\log T(t)) = \int kDdt$ or

$$T(t) = A_0 e^{kDT} \quad (10)$$

(ii) $\frac{X''(x)}{X(x)} = k$ implies that

$$X''(x) - kX(x) = 0 \quad (11)$$

With the resulting equations, there are cases to satisfy:

Case 1: k is positive, $k = \lambda^2$

$$X''(x) - \lambda^2 X(x) = 0$$

Solving the differential equation, the following is obtained:

$$X(x) = Ae^{\lambda x} + Be^{-\lambda x} \quad (12)$$

If x = 0, then B = -A. If x = L, then A = B = 0. Thus, there is a trivial solution because A and B are zeros.

Case 2: k = 0

$X'(x) = k$, k is a constant

Solving the differential equation, this yields another equation:

$$X(x) = A + Bx \quad (13)$$

If x = 0, then A = 0. If x = L, then A=B = 0 because L cannot be zero. Therefore, a trivial solution because A and B are zeros.

Case 3: k is negative, $k = -\lambda^2$

$$X(x) = Ae^{\lambda ix} + Be^{-\lambda ix} \quad \text{or} \quad (14)$$

$$X(x) = A\cos\lambda x + B\sin\lambda x$$

If x = 0, then A = 0. If x = L, then $X(L)$ = 0. This means that

$B\sin\lambda L$ = 0 with $\lambda L = n\pi$ or

$$\lambda = \frac{n\pi}{L}, \; , \; n = 1,2, \ldots \qquad (15)$$

Equation (14) can be reduced to

$$X(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) \qquad (16)$$

Substituting (10) and (16) to (5),

$$P(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n^2\pi^2}{L^2}\right)t} \qquad (17)$$

Given that $P(x,0) = f(x)$, the density function in (4), then

$$P(x,0) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) = f(x)$$

In order to find the value of B_n, the orthogonal relationship was used as follows,

$$\int_0^L \sin\left(\frac{n\pi}{L}x\right) \sin\left(\frac{m\pi}{L}x\right) dx \begin{cases} = 0, \text{when } n \neq m \\ \neq 0, \text{when } n = m \end{cases}$$

In evaluating,

$$\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx = \frac{B_n L}{2}, \text{ thus,}$$

$$B_n = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx \qquad (18)$$

Hence, in general,

$$P(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n^2\pi^2}{L^2}\right)t} \qquad (19)$$

Assuming that

$a < x < b, L = b - a, \text{and } a = \theta = \min(x).$

Then, P(θ ,t) = any constant and P(L,t) = any constant. Estimate by B_n integrating (18); $f(x)$ is needed which will differ across time where t = 1,2,3,.. and, B_1, B_2, B_3... are the infinite number of parameters.

Let $B_1 = \lambda_1, B_2 = \lambda_2, B_3 = \lambda_3$, where

$$\lambda_1 = \frac{1}{\text{average index in 2008}},$$
$$\lambda_2 = \frac{1}{\text{average index in 2013}}, \text{ and}$$
$$\lambda_3 = \frac{1}{\text{average index in 2018}}.$$

Thus, based from the histogram above,

$$\begin{aligned} f_1(x) &= \lambda_1 e^{-\lambda_1 x} \text{ (2008)}, \\ f_2(x) &= \lambda_2 e^{-\lambda_2 x} \text{ (2013)} \\ f_3(x) &= \lambda_3 e^{-\lambda_3 x} \text{ (2018)} \\ &\vdots \\ f_n(x) &= \lambda_n e^{-\lambda_n x} \text{ (nth)} \end{aligned}$$

With that,

$$B_n(x,t) = \lambda_1 e^{-\lambda_1 x} + \lambda_2 e^{-\lambda_2 x} + \lambda_3 e^{-\lambda_3 x} + \ldots + \lambda_n e^{-\lambda_n x}.$$

Considering the diffusion function, it can be inferred that when time goes to infinity, the value of $e^{-\left(\frac{n^2\pi^2}{L^2}\right)t}$ approaches to a normal curve distribution. The density function then is sinusoidal as time goes to infinity. With that, the behavior of the density function is characterized by the sine function, which goes up and down periodically. This equation demonstrates that hunger will persist even with better policies, programs, and projects to eradicate it. Hunger indices of various countries that are decreasing do not imply that they will be sustained until eradication because eventually these indices will increase again.

Conclusions

Hunger phenomenon is complex. The vast interconnectedness of its system agents makes it difficult to understand. With the many agents that interact on hunger, it is difficult to predict simply by looking at the individual interactions. Hunger incidences changes, reflective of every country's living condition and environment that are adaptive to the situation.

From the developed model, as time passes by, the distribution of the different countries' hunger indices will form a normal curve where a greater number of countries will fall in the average while a less number of countries will belong to the low and high hunger indices. With the model's sinusoidal behavior, hunger will persist and exist constantly. The hunger indices of various countries may decrease for now; however in time, it may increase tomorrow. Hunger will remain and will be experienced by humankind depending on the natural conditions.

Based on the results, people may not escape hunger. Natural calamities, human conflict, and war may result to hunger. Combating the hunger crisis will require more food, greater reforms in agriculture, more education and health sectors, and continuous monitoring of the food and nutrition situation. Yet, the irregularity of the indices provides prospect that the behavior of hunger may improve over time even if it persists. With diligence and commitment in implementing the plans and policies to achieve zero hunger, lesser hunger may be experienced.

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